Exam Nonlinear Optics

Thursday, March 31, 2016, 9.00-12.00 (room 5113.0201)

Give your name on each sheet.
On the first sheet, also give your student number
and the total number of sheets turned in.
Score = total points/2.7
Success!

Problem 1 (7 points, distributed according to (a,b,c,d) = (1, 1, 2.5, 2.5))

The Morse oscillator is a model for an anharmonic oscillator with the following restoring potential: $V(z)=m\beta\{1-\exp[-\alpha(z-z_0)]\}^2$, where z is the one-dimensional coordinate of the particle, m is its mass, and β, α, and z_0 are positive parameters. Consider a material where charged classical particles move in a Morse potential and also experience a damping force proportional to their velocity ($-2m\gamma v$). The density of oscillators is denoted *N*, the particle charge by *q*. In the problems below, always explain the quantities you introduce.

a) The Morse potential represents a well that obviously has a minimum at $z= z_0$. Give the power expansion of the potential around its equilibrium position up to and including terms of order x^3, where $x = z-z_0$. Use from now on x as the coordinate.

b) What is the eigenfrequency ω_0 of the oscillator (in the harmonic regime) in terms of the given parameters?

c) Derive the susceptibility $\chi^{(1)}(\omega)$.
Note: If you have been unable to solve problems a) and b), argue what is a physically acceptable form of the equation of motion for x in this particular problem (using the restoring force up to and including the lowest-order nonlinear term) and continue working with it.

d) Derive the susceptibility $\chi^{(2)}(\omega_1+\omega_2,\omega_1,\omega_2)$. What process does this susceptibility describe?

Problem 2 (4 points, distributed according to (a,b) = (2, 2))

Consider sum frequency generation, with all beams propagating in the z direction.

a) What is the wave vector mismatch? Explain the quantities you introduce. Argue why the signal is maximal if this mismatch equals zero?

b) What are the Manley-Rowe equations for this proces (no derivation needed!). Explain the symbols you introduce. What do these equations describe physically?

Problem 3 (8 points, distributed according to (a,b,c,d) = (3, 2, 2, 1))

Consider the molecular energy level diagram below. The transitions with nonzero transition dipoles are indicated by arrows; all other dipole matrix elements are zero (also all diagonal ones). The dephasing rate of the coherence |a><b| is given by γ_{ab}, and analogous for the other coherences. The molecule initially is in the ground state |a>.

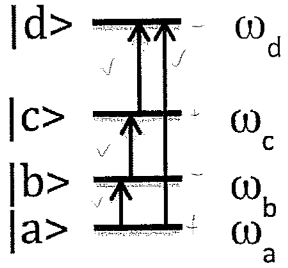
a) Draw a double-sided Feynman diagram for third-harmonic generation of a monochromatic laser beam with frequency ω that has a resonance at

$3\omega = \omega_{da}$. Calculate this diagram.

b) Does a diagram for third-harmonic generation exist with a resonance at $3\omega = \omega_{ca}$? If so, draw such a diagram; if not, argue why not.

c) Does a diagram for third-harmonic generation exist with a resonance at $2\omega = \omega_{db}$? If so, draw such a diagram; if not, argue why not.

d) Is the arrangement of transition dipoles possible for a centrosymmetric molecule? Explain your answer.



Problem 4 (8 points, distributed according to (a,b,c) = (4, 1.5, 2.5))

Consider an ensemble of two-level molecules in a host medium. As a result of inhomogeneity in the host, each molecule has a slightly different transition frequency. These frequencies are distributed according to a Gaussian distribution with mean ω_0 and standard deviation D. The coherence between the ground state and the excited state of each molecule decays exponentially with the same homogeneous dephasing rate γ.

a) Explain how one can use the two-pulse echo experiment to measure the homogeneous dephasing rate, even if $D >> \gamma$. Specifically: describe how the experiment is done and give an expression for the signal. You do not need to give a complete calculation; draw one of the Feynman diagrams that contributes to the two-pulse echo signal and present a clear analysis of the phase factors and relaxation factors and their averages to answer the question.

A more complex echo experiment is the three-pulse echo, in which one uses three pulses ($\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$), and one detects the signal in the direction $\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_1$ (so, to lowest order there are three interactions). All pulses have the same frequency; the length of the pulses obeys the same inequalities as in the two-pulse echo, and the time delay between pulse 1 and 2 is τ_1, while the one between pulses 2 and 3 is τ_2. One detects the signal a time t following pulse 3.

b) Show that the three-pulse echo experiment also allows for an echo type signal. To this end draw one Feynman diagram that generates an echo in this experiment.

c) Explain how one may use the three-pulse echo experiment to determine both the homogeneous dephasing rate γ and the population decay rate Γ of the excited state. Like in item a), a full calculation is not necessary.